**Methods**

**Stuart-Landau Model**

To investigate the effects of different inputs on a system of coupled oscillators, we used the Stuart-Landau model (Doelling and Assaneo, 2021; Strogatz, 2018; Hoppensteadt and Izhikevich, 2012):

where and are the two oscillators near Hopf-Andronov bifurcation (Strogatz, 2018). is the coupling parameter. For , the two oscillators oscillate synchronously with frequency whereas for , the system decays to a stable equilibrium. For all simulations, we set to 0.1 and to 0.05 and changed the which is the input to the system. We gave white, pink and brown noises (see below) scaled between input strengths 0.1, 1 and 10 to see their effect on synchronization. The differential equations were integrated with the Euler method with 0.1 milisecond steps for 100 seconds. First 50 seconds were discarded to get the steady state. The code to replicate the analyses can be found at https://github.com/duodenum96/anesthesia\_SL.

**Simulation of Colored Noise**

We used MATLAB’s dsp.ColoredNoise function from DSP System Toolbox to generate different inputs. Briefly, gaussian white noise is colored by multiplication with an autoregressive model of order 63 in the frequency domain to filter it’s power spectrum according to the power law (Kasdin, 1995). Three different input scenarios were simulated: white noise (), pink noise () and brown noise (). For each noise color, 200 simulations were performed.

**Quantification of Perturbation**

In the absence of any external stimulus, the two oscillators form a circle in the phase space. To compare the disturbance caused by noise, we took the absolute value of the difference in phase space after external stimulation. In figure XB left, we gave white, pink and brown noises to the system with logarithmically increasing noise strengths from 0.1 to 100 in 20 simulations. On right, we gave three different inputs strengths of 10, 30 and 50 with different noise colors from 0 (white noise) to 2 (brown noise).

**Results**

*Figure X.: A. The effect of different inputs on the Stuart-Landau model. We gave the model three different noise types with three different noise strengths each. As can be seen on the phase diagrams, input with a scale-free temporal structure can easily perturb a system even with low input strength whereas a white-noise input needs very high input strength to affect the system. Lines show mean across trials whereas shadings show standard deviations. B. The effect of the PLE of noise and noise strength on perturbation which was defined as the absolute value of the difference in the area of the circle in the phase space between perturbed and not perturbed states. The lines show mean and the shadings show standard error.*

We used Stuart-Landau model to investigate the effect of external perturbation on a system of coupled oscillators near criticality (Figure X). Even with low noise strength, the brown noise input with it’s high PLE value desynchronized the system even with a very low input strength. On the other hand, white noise needed very high input strength to make an effect. Pink noise stands between the tow signals in terms of both the PLE and needed input strength for perturbation.

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